Week 6 — Constraints and Real-World Models

Occasionally Binding Constraints & the Fischer–Burmeister Function

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The Constraint Problem

Many economic models feature inequality constraints:

- Borrowing limits: $a_{t+1} \geq \underline{a}$
- Non-negativity: $c_t \geq 0$, $k_t \geq 0$
- Collateral constraints: $b_t \leq \phi\, k_t$

These constraints **bind occasionally** — sometimes active, sometimes not.

Traditional KKT conditions create a discrete switch that is hard to differentiate.

Fischer–Burmeister Complementarity

The **FB function** provides a smooth reformulation:

$$\Phi(a, b) = a + b - \sqrt{a^2 + b^2}$$

Key property

$\Phi(a, b) = 0$ if and only if $a \geq 0$, $b \geq 0$, and $ab = 0$.

This is continuously differentiable — perfect for gradient-based training. It replaces the discrete if/else with a smooth function that autodiff can handle.

Consumption–Savings with Borrowing Constraint

$\max \sum \beta^t u(c_t)$ s.t. $a_{t+1} = (1+r)\,a_t + y_t - c_t$ and $a_{t+1} \geq \underline{a}$

Complementarity: either $a_{t+1} > \underline{a}$ (unconstrained) or $\mu_t > 0$ (constrained).

FB formulation: $\Phi(a_{t+1} - \underline{a},\ \mu_t) = 0$

Add this as an extra residual in the DEQN loss.

Architecture Design

- **Output activation**: softplus for consumption (ensures $c > 0$)
- **Width**: 3–4 hidden layers, 64–128 neurons each
- **Multiple outputs**: one head for c, another for μ (the multiplier)
- **Learning rate schedule**: cosine annealing helps convergence

Architecture search is mostly empirical — start simple, add complexity only if needed.

Summary

- Occasionally binding constraints are common and difficult
- Fischer–Burmeister smooths the complementarity condition
- DEQNs handle constraints naturally via extra residual terms
- Architecture choices (activations, depth) matter for convergence

Next week: Physics-Informed Neural Networks — continuous-time economics.