Week 8 — Heterogeneous Agents

The Curse of Dimensionality & Krusell–Smith with DEQNs

Dr Juan Zurita

Deep Learning for Macroeconomics
The University of Edinburgh · School of Economics

Why Heterogeneous Agents?

Representative-agent models miss:

- Wealth inequality and its aggregate effects
- Differential responses to policy across the distribution
- Precautionary saving and liquidity constraints

But HA models have a high-dimensional state: the entire wealth distribution Γ_t.

Traditional methods: approximate Γ_t with a few moments (Krusell & Smith, 1998).

Krusell–Smith (1998)

Key insight

Approximate the distribution by its **mean** (aggregate capital K). Agents' decision rules depend on (a_i, z_i, K, Z).

Algorithm:

1. Guess a law of motion: $\ln K' = b_0 + b_1 \ln K$ (conditional on Z)
2. Solve individual problems given this law
3. Simulate the economy and update b_0, b_1
4. Iterate until convergence

This works because $R^2 \approx 0.999$ — the mean is sufficient.

DEQNs for Heterogeneous Agents

Replace the parametric law of motion with a **neural network**:

$$K' = \mathcal{N}_\theta(K, Z)$$

Advantages:

- Can capture non-linear dynamics traditional regressions miss
- Extend to many moments without specifying functional forms
- Scale to models where "approximate aggregation" fails

Scheidegger and collaborators show DEQNs can handle state spaces with 50+ dimensions.

Young's Histogram Method

To simulate the distribution:

1. Discretise the individual state space into bins
2. Track the mass in each bin over time
3. Update masses using the policy function

This avoids simulating millions of individual agents. Combined with DEQNs, the entire solution is differentiable.

Summary

- HA models are essential for inequality and policy analysis
- The state space is the entire wealth distribution — very high-dimensional
- DEQNs overcome the curse of dimensionality
- Krusell–Smith with neural networks: more flexible, more scalable

Next week: surrogates, estimation, and climate economics.