

Week 9 — Surrogates, Estimation & Climate Economics

Gaussian Processes, SMM & the DICE Model

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Some models are too expensive to evaluate repeatedly (e.g. for estimation).

Surrogate (emulator): a cheap-to-evaluate approximation trained on a small number of model runs.

- **Gaussian processes:** non-parametric, with uncertainty quantification
- **Neural networks:** scale to higher dimensions
- **Polynomial chaos:** spectral approach for stochastic inputs

Gaussian Process Regression

A GP defines a distribution over functions: $f \sim \mathcal{GP}(m, k)$

- $m(x)$: mean function (often zero)
- $k(x, x')$: covariance kernel (e.g. squared exponential)

Given n evaluations of the expensive model, the GP provides:

1. A posterior mean (the surrogate prediction)
2. A posterior variance (uncertainty about the prediction)

The variance tells us **where to evaluate next** — active learning.

Simulated Method of Moments

Goal: estimate structural parameters θ by matching model moments to data moments.

$$\hat{\theta} = \arg \min_{\theta} [\mathbf{m}^{\text{data}} - \mathbf{m}^{\text{model}}(\theta)]' W [\mathbf{m}^{\text{data}} - \mathbf{m}^{\text{model}}(\theta)]$$

Problem: each evaluation of $\mathbf{m}^{\text{model}}(\theta)$ requires solving the full model.

Solution: replace the model with a trained surrogate $\Rightarrow$ fast evaluation.

The DICE Model

Nordhaus's **DICE** (Dynamic Integrated Climate–Economy):

- Couples economic growth with carbon emissions and climate dynamics
- Optimal abatement policy: how much to reduce emissions?
- High-dimensional when extended (regions, sectors, uncertainty)

Deep learning enables:

1. Solving richer versions of DICE with many state variables
2. Surrogate-based estimation of climate damage parameters
3. Uncertainty quantification via ensemble training

- Surrogates make expensive models tractable for estimation
- GPs provide uncertainty; neural networks provide scalability
- SMM + surrogates: estimate structural models efficiently
- Climate economics is a natural application of high-dimensional methods

Next week: frontiers and the big picture — when to use deep learning.