Week 3 — ARMA Models

Autoregressive and Moving Average Processes

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The AR Process

AR(1) — The Simplest Dynamic Model

$$y_t = c + \phi y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim WN(0, \sigma^2)$$

- **Stationary** iff $|\phi| < 1$
- Unconditional mean: $\mu = c/(1 - \phi)$
- Unconditional variance: $\sigma_y^2 = \sigma^2/(1 - \phi^2)$
- ACF: $\rho_k = \phi^k$ (geometric decay)
- PACF: cuts off after lag 1

AR(p) — General Autoregressive

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t$$

Stationarity condition: all roots of $1 - \phi_1 z - \phi_2 z^2 - \cdots - \phi_p z^p = 0$ lie outside the unit circle.

PACF of AR(p): cuts off after lag p.

The MA Process

MA(q) — Moving Average

$$y_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$$

- Always stationary (finite linear combination of white noise)
- ACF: cuts off after lag q
- PACF: decays geometrically
- Invertibility requires roots of $1 + \theta_1 z + \cdots$ outside unit circle

ARMA Models

ARMA(p,q)

$$y_t = c + \sum_{i=1}^{p} \phi_i y_{t-i} + \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}$$

Model	**ACF**	**PACF**
AR(p)	Decays	Cuts off at p
MA(q)	Cuts off at q	Decays
ARMA(p, q)	Decays	Decays

The Box-Jenkins Methodology

1. **Identification**: use ACF/PACF to determine p, q
2. **Estimation**: fit by MLE (or conditional least squares)
3. **Diagnostic checking**: are residuals white noise?
4. **Forecasting**: generate h-step-ahead predictions

Modern practice also uses **AIC/BIC** for step 1.

Summary

- **AR**: past *values* determine the present
- **MA**: past *shocks* determine the present
- **ARMA**: combines both
- ACF/PACF patterns $\Rightarrow$ model identification
- Next week: estimation, forecasting, and diagnostics