Week 5 — Exponential Smoothing & ETS

Simple, Holt, and Holt-Winters Methods

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Simple Exponential Smoothing

The SES Recursion

$$\hat{y}_{t+1|t} = \alpha y_t + (1-\alpha)\hat{y}_{t|t-1}, \quad \alpha \in [0,1]$$

Equivalently:

$$\hat{y}_{t+1|t} = \alpha \sum_{j=0}^{\infty} (1-\alpha)^j y_{t-j}$$

- α close to 1: fast adaptation, noisy forecast
- α close to 0: slow adaptation, smooth forecast
- Optimal for data with **no trend, no seasonality**

Holt's Linear Trend

Adding a Trend

Two equations:

$$\ell_t = \alpha y_t + (1-\alpha)(\ell_{t-1} + b_{t-1}) \qquad \text{(level)} \qquad (1)$$

$$b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1} \qquad \text{(trend)} \qquad (2)$$

Forecast: $\hat{y}_{t+h|t} = \ell_t + h \cdot b_t$

Damped trend variant

$\hat{y}_{t+h|t} = \ell_t + (\phi + \phi^2 + \cdots + \phi^h)b_t$

Prevents unrealistic long-run extrapolation.

Holt-Winters

Adding Seasonality

Additive:

$$\ell_t = \alpha(y_t - s_{t-m}) + (1-\alpha)(\ell_{t-1} + b_{t-1}) \tag{3}$$

$$b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1} \tag{4}$$

$$s_t = \gamma(y_t - \ell_{t-1} - b_{t-1}) + (1-\gamma)s_{t-m} \tag{5}$$

Forecast: $\hat{y}_{t+h|t} = \ell_t + hb_t + s_{t+h-m}$

Multiplicative: seasonal component multiplies level.

The ETS Framework

Error, Trend, Seasonal (ETS)

Systematic taxonomy:

- **Error**: Additive (A) or Multiplicative (M)
- **Trend**: None (N), Additive (A), Damped (Ad)
- **Seasonal**: None (N), Additive (A), Multiplicative (M)

ETS(A,A,A) = additive Holt-Winters.
ETS(M,Ad,M) = damped-trend multiplicative.

All can be estimated by MLE and compared via AIC.

Summary

- **SES**: no trend, no season — one parameter α
- **Holt**: adds trend — two parameters α, β^*
- **Holt-Winters**: adds season — three parameters α, β^*, γ
- **ETS**: unifying framework; model selection via AIC
- Next week: VAR models for multivariate forecasting