Week 6 — Multiple Time Series & VAR Models

Vector Autoregressions, IRFs, and Granger Causality

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The VAR Model

From Univariate to Multivariate

Why go multivariate?

- Variables interact: GDP affects unemployment affects consumption
- Omitting relevant variables biases forecasts
- We want to trace **dynamic interactions** (impulse responses)

VAR(p) Specification

For $\mathbf{y}_t = (y_{1t}, \ldots, y_{nt})'$:

$$\mathbf{y}_t = \mathbf{c} + \mathbf{A}_1 \mathbf{y}_{t-1} + \cdots + \mathbf{A}_p \mathbf{y}_{t-p} + \mathbf{u}_t$$

where $\mathbf{u}_t \sim N(\mathbf{0}, \Sigma)$.

- Each equation is a **multivariate regression**
- Estimated by **OLS equation by equation**
- Parameters: $n^2 p + n$ — grows fast!

Impulse Response Functions

IRFs

An IRF traces the effect of a one-unit shock in variable j on variable i over time.

Write the VAR in MA(∞) form:

$$\mathbf{y}_t = \boldsymbol{\mu} + \sum_{s=0}^{\infty} \Phi_s \mathbf{u}_{t-s}$$

IRF: $\frac{\partial y_{i,t+h}}{\partial u_{j,t}} = [\Phi_h]_{ij}$

Identification problem: shocks may be correlated (Σ not diagonal).
Common solution: **Cholesky decomposition** of Σ.

Granger Causality

Granger Causality

x Granger-causes y if past values of x help predict y beyond y's own past.

Test: in the equation for y_t, test H_0: all coefficients on lagged x are zero.

Caution

Granger causality $\neq$ true causality. It is about *predictive content*, not causal mechanisms.

VAR Forecasting

- Iterate the VAR forward: $\hat{\mathbf{y}}_{T+h|T} = \mathbf{c} + \sum_{i=1}^{p} \mathbf{A}_i \hat{\mathbf{y}}_{T+h-i|T}$
- Lag order selection: AIC, BIC, HQ
- Forecast uncertainty: bootstrap or asymptotic intervals

Next week: state space models and the Kalman filter.