Week 1 — Mathematical Foundations

Sets, Functions & Linear Algebra

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Roadmap

This course builds the mathematical theory of optimisation for economics.

Today: the toolkit — sets, vectors, matrices, eigenvalues, positive definiteness.

Why? Every optimisation result rests on these foundations.

Sets in $\mathbb{R}^n$

- **Open set:** every point has a neighbourhood inside the set
- **Closed set:** contains all its limit points
- **Compact set:** closed and bounded (Heine–Borel)

Extreme Value Theorem

A continuous function on a compact set attains its maximum and minimum.

Vectors and Matrices

A **vector** $\mathbf{x} \in \mathbb{R}^n$; a **matrix** $A \in \mathbb{R}^{m \times n}$.

Key operations: dot product, matrix multiplication, transpose, inverse, determinant.

In Python: `numpy` handles all of these efficiently.

Eigenvalues

For square A: $A\mathbf{v} = \lambda\mathbf{v}$

λ = eigenvalue, $\mathbf{v}$ = eigenvector.

Definiteness (symmetric A)

- All $\lambda > 0 \Rightarrow$ **positive definite**
- All $\lambda < 0 \Rightarrow$ **negative definite**
- Mixed signs $\Rightarrow$ **indefinite**

Why Definiteness Matters

At a critical point $\nabla f = 0$:

- Hessian **negative definite** $\Rightarrow$ local **maximum**
- Hessian **positive definite** $\Rightarrow$ local **minimum**
- Hessian **indefinite** $\Rightarrow$ **saddle point**

This is the second-order condition we formalise next week.

Summary

- Sets, functions, vectors, matrices — the language of optimisation
- Eigenvalues classify matrices as positive/negative definite
- Definiteness of the Hessian $\rightarrow$ nature of critical points
- NumPy makes these computations trivial in Python

Next week: Gradients, Hessians, and convexity.