

Week 6 — Convex Optimisation

Theory, Duality & CVXPY

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Why Convexity?

The Convex Optimisation Guarantee

Every local minimum is **global**. Efficient algorithms exist.

Many economic models are naturally convex: cost minimisation, portfolio optimisation, regression.

Lagrangian Duality

Primal: $\min f(\mathbf{x})$ s.t. $g_i \leq 0$

Dual function: $d(\mu) = \inf_{\mathbf{x}} \{f(\mathbf{x}) + \sum \mu_i g_i(\mathbf{x})\}$

Weak duality: $d^* \leq p^*$ always

Strong duality: $d^* = p^*$ for convex problems (Slater's condition)

CVXPY: a Python library for **disciplined convex programming**.

```
import cvxpy as cp
x = cp.Variable(n)
prob = cp.Problem(cp.Minimize(cp.sum_squares(A@x-b)),
                  [x >= 0])
prob.solve()
```

Automatically checks convexity and selects the right solver.

Summary

- Convexity guarantees global optimality
- Strong duality: primal value = dual value
- CVXPY makes convex optimisation accessible in Python
- **Next week:** linear programming — a special case with rich structure