

Week 7 — Linear Programming

The Simplex Method & Economic Models

Dr Juan Zurita

Optimisation & Mathematical Methods for Economics
The University of Edinburgh · School of Economics

$$\min \mathbf{c}^T \mathbf{x} \text{ s.t. } A\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0$$

Fundamental Theorem of LP

If an LP has an optimal solution, there is one at a **vertex** of the feasible polyhedron.

The Simplex Method

Dantzig (1947): move from vertex to vertex, improving the objective at each step.

- Worst case: exponential; in practice: very fast
- The most successful algorithm in operations research

LP Duality & Shadow Prices

Every LP has a **dual**. Optimal primal value = optimal dual value.

Dual variables = **shadow prices**: how much the objective improves per unit relaxation of each constraint.

In economics: the marginal value of an additional unit of a scarce resource.

- **Input–output models:** Leontief's production planning
- **Transportation:** minimise shipping cost across a network
- **Diet problem:** minimise food cost meeting nutrition targets
- **Production planning:** allocate resources across products

In Python: `scipy.optimize.linprog` or PuLP.

- LP: optimise linear objective over linear constraints
- Simplex moves along vertices; solution is always at a corner
- Duality provides shadow prices for economic interpretation
- **Next week:** dynamic optimisation — optimising over time