

Module 1 — The Invisible Hand

Adam Smith and Market Equilibrium

Dr Juan Zurita

From Smith to Simulation

The University of Edinburgh · School of Economics

Part I — The History

- March 1776: a retired professor of moral philosophy publishes *An Inquiry into the Nature and Causes of the Wealth of Nations*.
- **Adam Smith** (1723–1790) had spent ten years writing it — mostly in his mother's house in Kirkcaldy.
- The book that **founded modern economics**.

Edinburgh Connection

Smith attended the University of Edinburgh before Oxford (which he found “vastly inferior”). He spent his last years at **Panmure House** on the Canongate — you can still visit it today.

What Smith Was Arguing Against

Mercantilism

The dominant economic doctrine of 1776:

- National wealth = hoarding gold and silver.
- Governments set tariffs, granted monopolies, banned exports.
- The economy was something to be *directed from above*.

Smith's radical claim: government direction is not only **unnecessary** but **counterproductive**.

The Invisible Hand

“It is not from the benevolence of the butcher, the brewer, or the baker that we expect our dinner, but from their regard to their own interest.”

— Adam Smith, *The Wealth of Nations*, Book I, Ch. 2

- Individuals pursuing self-interest $\Rightarrow$ socially beneficial outcomes.
- No one plans the economy, yet bread gets baked, shoes get made, goods reach those who want them.
- The mechanism: **prices and markets** coordinate decisions.

The Scottish Enlightenment

Smith was part of one of the most extraordinary intellectual circles in history:

David Hume Philosopher, historian, Smith's closest friend. Radical empiricism shaped Smith's method.

Joseph Black Chemist who discovered latent heat, right here at Edinburgh.

James Hutton Geologist who realised the Earth was unimaginably old.

Adam Ferguson Coined the phrase "civil society."

Shared commitment: understand the world through **observation and systematic reasoning**, not appeals to authority.

Supply, Demand, and the Natural Price

Smith distinguished:

Natural Price

What a good costs to produce (wages, rent, profit at “ordinary” rates).

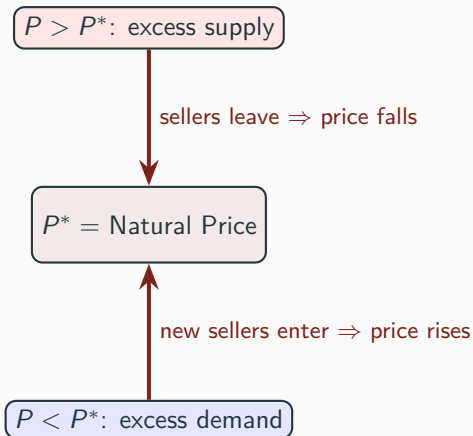
Market Price

What the good actually sells for, driven by the balance of supply and demand.

“The natural price . . . is, as it were, the central price, to which the prices of all commodities are continually gravitating.”

— WN, Book I, Ch. 7

The Gravitational Logic



The market **self-corrects** — without anyone directing it.
This is the invisible hand at work.

Part II — The Computation

Setting Up

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import fsolve
```

- numpy — numerical arrays and operations
- matplotlib — plotting
- fsolve — numerical root-finder
(the computational equivalent of Smith's “gravitating” price)

Supply and Demand as Functions

Edinburgh's oat market, c. 1770:

$$Q^D(P) = a - bP = 200 - 4P \quad (\text{Demand})$$

$$Q^S(P) = c + dP = 20 + 3P \quad (\text{Supply})$$

- Demand slopes down: higher price $\Rightarrow$ fewer buyers.
- Supply slopes up: higher price $\Rightarrow$ more sellers.
- Where do they cross? That is the equilibrium.

Defining the Market in Python

```
# Parameters for Edinburgh's oat market
a, b = 200, 4    # demand:  $Q_D = a - b*P$ 
c, d = 20, 3     # supply:  $Q_S = c + d*P$ 

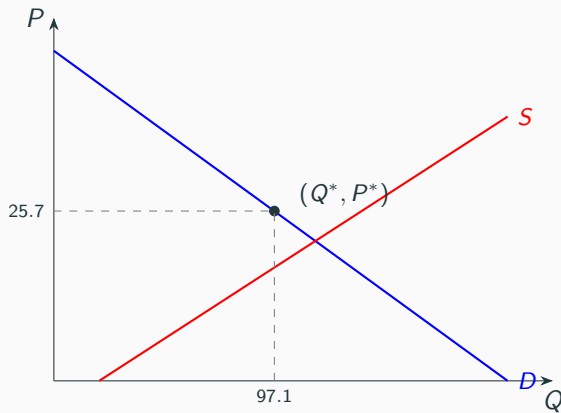
def demand(P):
    return a - b * P

def supply(P):
    return c + d * P
```

At $P = 20$ shillings:

- $Q^D = 200 - 4(20) = 120$ bushels demanded
- $Q^S = 20 + 3(20) = 80$ bushels supplied
- **Shortage** of 40 bushels $\Rightarrow$ price rises

Plotting the Market



Convention (Marshall): price on y -axis, quantity on x -axis.

$$ED(P) = Q^D(P) - Q^S(P)$$

- $ED(P) > 0$: **shortage** — buyers want more than sellers offer. Price tends to *rise*.
- $ED(P) < 0$: **surplus** — sellers offer more than buyers want. Price tends to *fall*.
- $ED(P) = 0$: **equilibrium**.

Smith's gravitational logic, expressed as a function whose **root** gives us P^* .

Finding Equilibrium with fsolve

Analytical (for linear case):

$$a - bP = c + dP \implies P^* = \frac{a - c}{b + d} = \frac{180}{7} \approx 25.71$$

Numerical (works for *any* market):

```
def excess_demand(P):  
    return demand(P) - supply(P)  
  
P_star = fsolve(excess_demand, x0=10)[0]  
Q_star = demand(P_star)  
print(f"P* = {P_star:.4f}, Q* = {Q_star:.4f}")
```

fsolve takes a function and a starting guess $\Rightarrow$ finds where $f(x) = 0$.

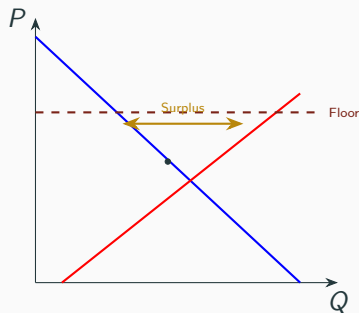
Numerical and analytical solutions agree to machine precision.

Price Floors: When Government Overrules the Invisible Hand

Suppose Edinburgh's town council sets a **price floor** of 35 shillings (above $P^* = 25.7$):

- $Q^D(35) = 200 - 4(35) = 60$
- $Q^S(35) = 20 + 3(35) = 125$
- Surplus: $125 - 60 = 65$ unsold bushels

Smith's warning: the government, trying to help farmers, creates **a pile of unsold oats** and **reduces** the quantity traded.



A Nonlinear Market: Why We Need Computers

Real markets are rarely linear:

$$Q^D(P) = \frac{500}{1 + 0.1 P^{1.5}}$$

$$Q^S(P) = 10 \ln(1 + P)$$

No algebraic solution exists — but `fsolve` handles it:

```
def excess_demand_nl(P):  
    return 500 / (1 + 0.1*P**1.5) - 10*np.log(1 + P)  
  
P_star_nl = fsolve(excess_demand_nl, x0=20)[0]  
# P* = 22.61, Q* = 32.60
```

The invisible hand works the same way whether the curves are straight lines or complicated nonlinear functions.

Simulating the Invisible Hand

Smith: the market price “gravitates” toward equilibrium.

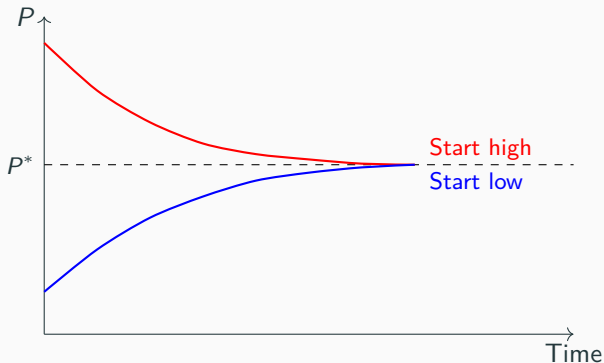
Price adjustment rule:

$$P_{t+1} = P_t + \lambda \cdot ED(P_t)$$

```
def simulate_market(P0, lam, T, ed_func):  
    prices = np.empty(T)  
    prices[0] = P0  
    for t in range(1, T):  
        prices[t] = prices[t-1] + lam * ed_func(prices[t-1])  
    return prices
```

- Shortage ($ED > 0$) $\Rightarrow$ price rises
- Surplus ($ED < 0$) $\Rightarrow$ price falls
- λ controls the speed of adjustment

Convergence to Equilibrium



No matter where the price starts, it converges to equilibrium.

The market corrects itself — shortages push prices up, surpluses push them down.

Part III — Exercises & Discussion

Exercise 1: A Tax on Oats

The Scottish Parliament imposes a **per-unit tax** $\tau = 5$ shillings, paid by sellers.

$$Q_{\text{tax}}^S(P) = c + d(P - \tau)$$

1. Define `supply_tax(P, tau=5)` and a new excess demand function.
2. Use `fsolve` to find the new equilibrium.
3. Who bears more of the tax — buyers or sellers?
4. Plot old vs. new supply curves with both equilibria.

Exercise 2: The Corn Laws

Two-country trade model (Britain vs. France):

	Britain	France
Demand	$Q^D = 300 - 5P$	$Q^D = 200 - 3P$
Supply	$Q^S = 50 + 2P$	$Q^S = 30 + 4P$

1. Find each country's autarky equilibrium price.
2. Which country has the lower price? What does Smith predict?
3. Find the world equilibrium price (sum of excess demands = 0).
4. At the world price, who imports and who exports?

Exercise 3: Does the Starting Guess Matter?

1. Try the nonlinear excess demand with starting guesses $P_0 = 1, 10, 50, 100, 500$. Do all converge to the same root?
2. Try the pathological function:

$$ED(P) = \sin(P) - 0.1P + 1$$

Multiple roots! Different starting guesses $\Rightarrow$ different solutions.

3. Connection to Smith: is the “natural price” always unique?

Lesson

Computational methods require care — always check your results and try multiple starting points.

Key Takeaways

What We Learned This Week

1. Adam Smith (1776) showed that **self-interested behaviour**, channelled through markets, can produce socially beneficial outcomes.
2. The **invisible hand** is the tendency of market prices to gravitate toward equilibrium.
3. We can **compute** equilibria by finding roots of excess demand functions using `fsolve`.
4. Government interventions (price floors, taxes) can be modelled and their welfare effects quantified.
5. Numerical methods handle **nonlinear** markets where algebra fails.

Further Reading

- Smith, A. *The Wealth of Nations* (1776), Book I, Chs. 1–7.
Free at <https://www.econlib.org/library/Smith/smWN.html>
- Heilbroner, R. *The Worldly Philosophers*, Ch. 3.
- Broadie, A. *The Scottish Enlightenment*.
- Phillipson, N. *Adam Smith: An Enlightened Life*.

Next week: The Division of Labour and the Pin Factory —
Smith's most famous example, and the logic of specialisation and trade.