Module 3 — Malthus and Ricardo

Population, Land, and the Limits to Growth

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From Smith to Simulation

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Part I — The History

London, 1798

- Twenty-two years after Smith's optimistic vision, a young English clergyman publishes *An Essay on the Principle of Population*.
- **Thomas Robert Malthus** (1766–1834) argued that humanity was trapped.
- One of the most influential — and controversial — books in the history of social science.

The Core Argument

Population, when unchecked, grows **geometrically**. Food production can only grow **arithmetically**. Therefore, population will always tend to outrun the food supply.

The Darkest Idea in Economics

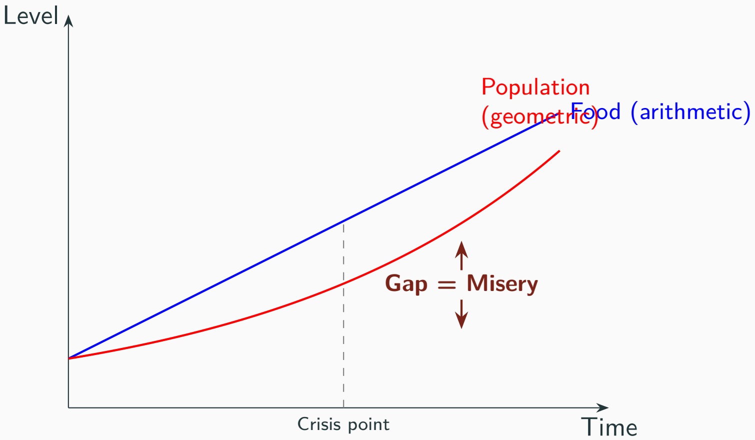
"The power of population is indefinitely greater than the power in the earth to produce subsistence for man."

— Thomas Malthus, *Essay on the Principle of Population* (1798), Ch. 1

Malthus's logic was brutally simple:

1. **Population** grows geometrically (exponentially) — each generation is larger than the last.
2. **Food production** grows only arithmetically (linearly) — there is only so much land.
3. The result is misery: famine, disease, and war keep population in check.

Geometric vs Arithmetic Growth



Population growth eventually **outstrips** food production.

The gap is closed by what Malthus called “positive checks”: famine, disease, war.

The Dismal Science

Why Economics Earned Its Nickname

Thomas Carlyle called economics *“the dismal science”* — the phrase was aimed at the gloomy predictions of Malthus and his followers.

- Smith (1776) was **optimistic**: markets create wealth for all.
- Malthus (1798) was **pessimistic**: population devours any gains.
- Wages are forever pulled back to subsistence.
- No amount of charity or good intentions can change this — it is a law of nature.

For most of human history, **Malthus was right**.

Was Malthus Right? The Malthusian Trap

- For **thousands of years** before the Industrial Revolution, living standards barely changed.
- When agricultural productivity improved — a new crop, a better plough — the result was not richer people but *more* people.
- Population expanded until it pressed against the food supply, and wages returned to subsistence.
- Gregory Clark (*A Farewell to Alms*, 2007): the average English person in 1800 was no better off than in 100,000 BC.

The Trap

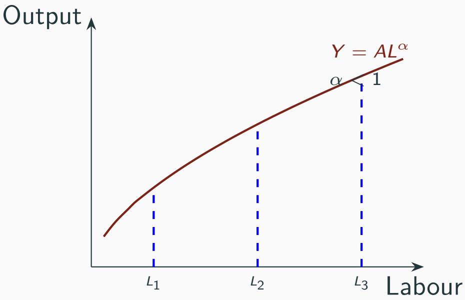
Any surplus is eaten up (literally) by population growth. Better technology $\Rightarrow$ more people, **not** richer people.

Then, around 1800, something extraordinary happened...

David Ricardo and Diminishing Returns

Malthus's friend and sparring partner, **David Ricardo** (1772–1823), formalised the mechanism:

- As more labour is applied to **fixed land**, each additional worker produces *less* additional output.
- First farmers clear the best land; then the second-best; eventually, rocky hillsides.
- This is why food production can't keep up — not because farming doesn't improve, but because the marginal improvement gets smaller.



Ricardo's Income Distribution

Ricardo showed how diminishing returns determined the **distribution of income**:

Workers

Receive **wages** $=$ marginal product of labour.

As population grows, wages *fall* toward subsistence.

Landlords

Receive **rents** $= Y - wL$.

As population grows, rents *rise* — landlords capture an increasing share.

Population growth $\Rightarrow$ workers *worse off*, landlords *better off*.

A grim picture — and Marx would later build on it.

Edinburgh and the Scottish Connection

- Malthus's *Essay* was partly a response to Enlightenment optimists (Condorcet, Godwin) who believed humanity was perfectible.
- Malthus — educated in the Scottish tradition of empirical argument — responded with **data and logic** rather than hope.
- **Dugald Stewart**, who held Adam Smith's old chair of moral philosophy at Edinburgh, disseminated both Smith's and Malthus's ideas to the next generation.
- The Scottish Enlightenment's commitment to following evidence wherever it leads, even to uncomfortable conclusions, is visible throughout Malthus's work.

Part II — The Computation

Setting Up

```
import numpy as np
import matplotlib.pyplot as plt
```

- `numpy` — numerical arrays and operations
- `matplotlib` — plotting

We will build and simulate the Malthusian model step by step, then watch it **reproduce ten thousand years of economic history**.

The Production Function: Diminishing Returns

Following Ricardo, we model food production with a **Cobb–Douglas production function** where land is fixed:

$$Y = A \cdot L^{\alpha}$$

- Y = total food output
- L = population (= labour force in this pre-industrial world)
- A = productivity parameter (quality of seeds, techniques)
- $\alpha \in (0, 1)$ captures diminishing returns

The crucial feature: $\alpha < 1$ means doubling the population does *not* double food production.

This is Ricardo's diminishing returns, expressed mathematically.

Production Function in Python

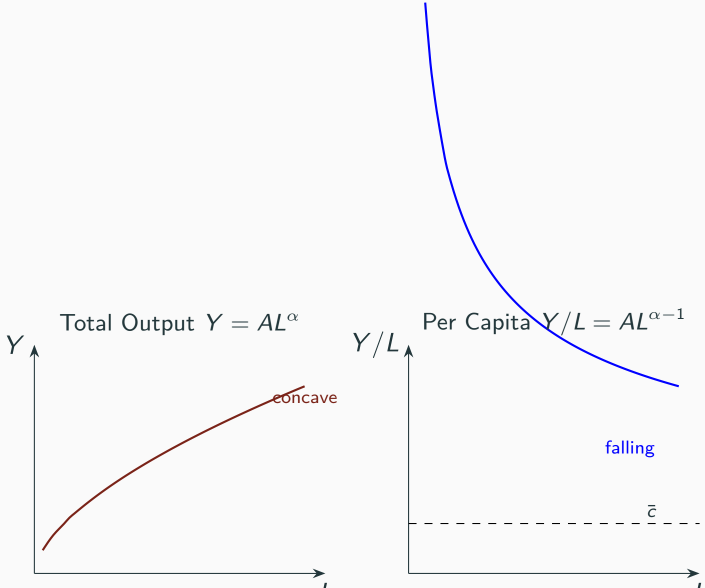
```python
# Parameters
A = 10        # productivity
alpha = 0.6   # diminishing returns (less than 1)

def production(L, A=A, alpha=alpha):
    """Total food output given population L."""
    return A * L**alpha

def output_per_capita(L, A=A, alpha=alpha):
    """Food per person = Y/L = A * L^(alpha-1)."""
    return A * L**(alpha - 1)
```

Total output rises with L…

…but **output per person** falls.

This is the essence of the Malthusian trap.

Diminishing Returns: The Picture



Population Dynamics: The Malthusian Mechanism

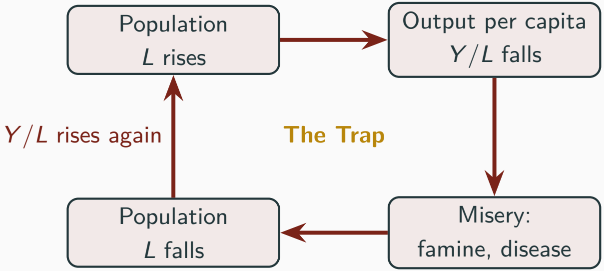
Malthus's key insight: population growth *responds* to living standards.

$$L_{t+1} = L_t \cdot \left(1 + g\left(y_t - \bar{c}\right)\right)$$

where $y_t = Y_t / L_t$ is food per person.

- $y > \bar{c}$: plenty of food $\Rightarrow$ population **grows** (more children, fewer deaths).
- $y < \bar{c}$: famine $\Rightarrow$ population **shrinks** (fewer children, more deaths).
- $y = \bar{c}$: population is **stable** — the Malthusian steady state.

The Feedback Loop

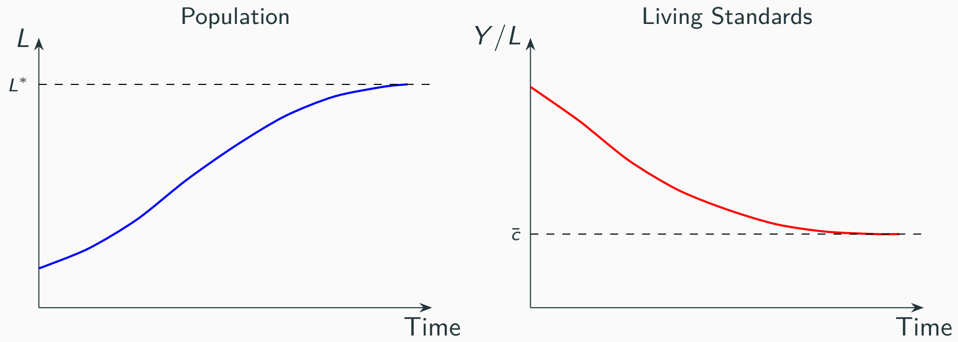


A self-correcting system — but one that corrects through **suffering**.

Simulating the Malthusian Model

```
c_bar = 1.5      # subsistence consumption
g = 0.02         # population growth sensitivity
T = 300          # number of periods
L0 = 50          # initial population

def simulate_malthus(L0, A, alpha, c_bar, g, T):
    L = np.empty(T)
    y = np.empty(T)
    L[0] = L0
    y[0] = A * L0**(alpha - 1)

    for t in range(1, T):
        growth_rate = g * (y[t-1] - c_bar)
        L[t] = max(L[t-1] * (1 + growth_rate), 1)
        y[t] = A * L[t]**(alpha - 1)
    return L, y

L_sim, y_sim = simulate_malthus(L0, A, alpha, c_bar, g, T)
```

Convergence to the Steady State



Population grows until output per capita falls to subsistence, then stabilises.

Living standards are trapped — exactly as Malthus predicted.

Finding the Steady State Analytically

At the Malthusian steady state, output per capita equals subsistence:

$$\frac{Y}{L} = A \cdot L^{\alpha-1} = \bar{c}$$

Solving for L^*:

$$\boxed{L^* = \left(\frac{A}{\bar{c}}\right)^{\frac{1}{1-\alpha}}}$$

- Higher productivity $A \Rightarrow$ **larger** steady-state population.
- But living standards are always $\bar{c}$ — technology changes *how many* people live at subsistence, not *how well*.

Verifying the Steady State

```python
def malthus_steady_state(A, alpha, c_bar):
    """Analytical steady-state population."""
    return (A / c_bar) ** (1 / (1 - alpha))

L_star = malthus_steady_state(A, alpha, c_bar)
print(f"Analytical L* = {L_star:.2f}")
print(f"Simulation L*  = {L_sim[-1]:.2f}")
```

A	L^*	Living standard
8	65.1	$\bar{c} = 1.5$
10	128.3	$\bar{c} = 1.5$
15	407.6	$\bar{c} = 1.5$
20	928.3	$\bar{c} = 1.5$

Better technology $\Rightarrow$ bigger population, **same** living standard.

The Cruel Irony

Does Better Technology Help?

Suppose someone invents a better plough (higher A). You might expect everyone to become richer. But:

1. Higher $A \Rightarrow$ output per capita temporarily rises above $\bar{c}$.
2. Population grows in response.
3. Output per capita falls back to $\bar{c}$.
4. The only lasting effect is a larger population.

This explains why China and India had the world's largest populations for centuries but not the highest living standards — productive agriculture translated into *more mouths* rather than *fuller plates*.

Simulating the Cruel Irony

```
for A_val, label in [(8,  'A=8  (poor tech)'),
                     (10, 'A=10 (baseline)'),
                     (15, 'A=15 (better plough!)')]:
    L_s, y_s = simulate_malthus(L0, A_val, alpha, c_bar, g, T)
    plt.plot(y_s, label=label)

plt.axhline(c_bar, color='k', linestyle='--', label='Subsistence')
plt.legend()
plt.title('Living Standards for Different Technologies')
plt.show()
```

All three economies converge to the **same** subsistence level.

Better technology $\Rightarrow$ more people, **not** richer people.

The Wage Function

In a competitive labour market, the wage equals the **marginal product of labour**:

$$w = MPL = \alpha \cdot A \cdot L^{\alpha-1}$$

Total wages:

$$wL = \alpha \cdot Y$$

Total rents (to landlords):

$$Y - wL = (1-\alpha) \cdot Y$$

As population grows:

- The *wage per worker* w **falls** (diminishing returns).
- Total *rents* $(1-\alpha)Y$ **rise** (more output, landlords' share is constant).
- Ricardo's conclusion: population growth benefits landlords at the expense of workers.

Breaking the Trap: The Industrial Revolution

The Malthusian model describes the pre-industrial world perfectly.
But after 1800, something changed.

Model technological progress as continuous productivity growth:

$$A_t = A_0 \cdot e^{\gamma t}$$

- If γ is small: population absorbs the gains $\Rightarrow$ **still trapped**.
- If γ is large enough: productivity outpaces population $\Rightarrow$ escape!

The question of *why* this happened — why in Britain, why around 1800 — remains one of the most debated questions in economic history.

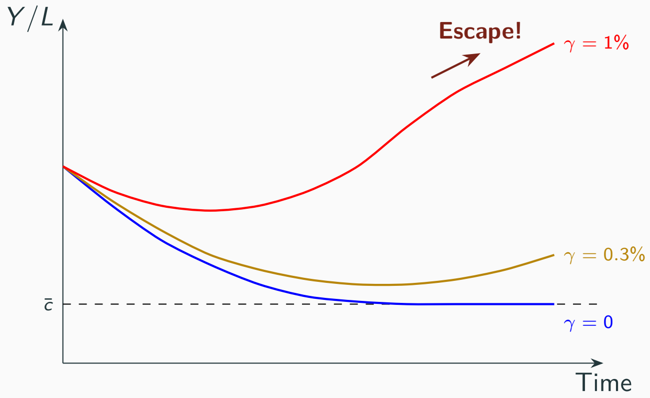
Simulating the Escape

```python
def simulate_with_growth(L0, A0, alpha, c_bar, g, gamma, T):
    L = np.empty(T);  y = np.empty(T);  A_t = np.empty(T)
    L[0], A_t[0] = L0, A0
    y[0] = A0 * L0**(alpha - 1)
    for t in range(1, T):
        A_t[t] = A0 * np.exp(gamma * t)
        growth_rate = g * (y[t-1] - c_bar)
        L[t] = max(L[t-1] * (1 + growth_rate), 1)
        y[t] = A_t[t] * L[t]**(alpha - 1)
    return L, y, A_t
```

Three scenarios:

- $\gamma = 0$: no growth (pre-1800) $\Rightarrow$ trapped
- $\gamma = 0.003$: slow growth $\Rightarrow$ partially escapes
- $\gamma = 0.01$: Industrial Revolution $\Rightarrow$ full escape

The Great Escape



With fast enough technological progress, living standards **break free from subsistence**.

For 10,000 years the world was stuck. Then, in a few decades, everything changed.

Part III — Exercises & Discussion

Exercise 1: The Black Death (1348)

The Black Death killed roughly **one-third** of Europe's population. The Malthusian model makes a clear prediction.

1. Simulate the Malthusian model for 500 periods with $L_0 = 200$, $A = 10$, $\alpha = 0.6$, $\bar{c} = 1.5$, $g = 0.02$. Let it reach steady state.
2. At period 250, kill one-third of the population (multiply L by $2/3$). Continue the simulation.
3. What happens to living standards *immediately* after the plague? In the *long run*?
4. In 2–3 sentences, explain why a catastrophe that kills millions can *improve* living standards for survivors.

Historical evidence

Real wages in England roughly **doubled** after the Black Death.

Exercise 2: Ricardo's Income Distribution

With $Y = A \cdot L^{\alpha}$, the wage is $w = \alpha A L^{\alpha-1}$ and total rents are $(1-\alpha)Y$.

1. Compute and plot wage w and total rents $(1-\alpha)Y$ as functions of L for $L \in [10, 500]$. Use $A = 10$, $\alpha = 0.6$.
2. As population doubles from 100 to 200, what happens to individual wages? To total rents? Who benefits?
3. Compute and plot the **rent-to-wage ratio**:

$$\frac{(1-\alpha)\,Y}{\alpha \cdot A \cdot L^{\alpha-1}} = \frac{(1-\alpha)}{\alpha} \cdot L$$

   What happens as population grows?

Exercise 3: Carrying Capacity

Modify the model to include an environmental **carrying capacity** K:

$$Y = A \cdot L^{\alpha} \cdot \left(1 - \frac{L}{K}\right)$$

When L approaches K, output drops to zero.

1. Simulate with $K = 500$, $A = 10$, $\alpha = 0.6$, $L_0 = 50$.
2. What happens to the steady-state population and living standards compared to the basic model?
3. Add technological progress ($\gamma = 0.005$). Does the economy still escape the trap? What is different?

Modern relevance

Some argue Malthusian dynamics apply today — not for food, but for energy, water, and the atmosphere's capacity to absorb CO_2.

Quiz

Quiz: Conceptual Questions

1. Malthus argued population grows geometrically while food grows arithmetically. In modern terms:
   (a) Population linear, food exponential
   (b) Population exponential, food linear
   (c) Both at the same rate
   (d) Both exponential at different rates
2. A one-time improvement in productivity (higher A) leads to:
   (a) Permanently higher living standards
   (b) Temporarily higher, then back to subsistence with larger population
   (c) Lower living standards
   (d) No change
3. Ricardo's "diminishing returns" means:
   (a) Profits fall to zero
   (b) Each additional worker on fixed land produces less additional output
   (c) Wages always decrease
   (d) Trade produces diminishing benefits

Quiz: Computational Questions

6. In $Y = A \cdot L^{0.6}$, if population doubles from 100 to 200, total output:
   (a) Exactly doubles
   (b) More than doubles
   (c) Increases by $2^{0.6} \approx 1.52$ (less than doubles)
   (d) Stays the same
7. The steady-state population is $L^* = (A/\bar{c})^{1/(1-\alpha)}$. If A doubles, L^*:
   (a) Doubles
   (b) More than doubles ($2^{2.5} \approx 5.66$ when $\alpha = 0.6$)
   (c) Less than doubles
   (d) Depends on α
8. If the Black Death halves the population, the model predicts living standards will:
   (a) Fall and stay low
   (b) Rise immediately, then gradually return to subsistence
   (c) Stay unchanged
   (d) Rise permanently

Key Takeaways

What We Learned This Week

1. Malthus (1798) argued that **population growth** always outpaces food production, trapping humanity at **subsistence**.
2. Ricardo formalised **diminishing returns**: each worker added to fixed land produces less additional output ($Y = AL^{\alpha}$, $\alpha < 1$).
3. We can **simulate** the Malthusian trap: population grows until $Y/L = \bar{c}$, then stabilises.
4. The cruel irony: better technology produces **more people**, not richer people.
5. The **Industrial Revolution** broke the trap — sustained technological progress ($\gamma > 0$) can outpace population growth.
6. For 99% of human history, Malthus was right. Understanding *why* his model stopped working around 1800 remains a central question.

Further Reading

- Malthus, T.R. *An Essay on the Principle of Population* (1798), Ch. 1.
  Free at `https://www.econlib.org/library/Malthus/malPlong.html`
- Clark, G. *A Farewell to Alms: A Brief Economic History of the World* (2007), Chs. 1–2.
- Heilbroner, R. *The Worldly Philosophers*, Ch. 4
  ("The Gloomy Presentiments of Parson Malthus and David Ricardo").
- Galor, O. *Unified Growth Theory* (2011) — the modern framework for the escape from the Malthusian trap.

Next week: Marx and the Machinery Question —
capital accumulation, the labour share, and whether technology helps or hurts workers.