Module 4 — Marx and the Machinery Question

Capital, Surplus Value, and the Labour Share

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From Smith to Simulation
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Part I — The History

The Dark Side of the Pin Factory

- Adam Smith marvelled at the pin factory's productivity.
- But by the 1840s, the Industrial Revolution raised an uncomfortable question: who benefits?
- Factories produced more than ever, yet workers laboured 14-hour days in dangerous conditions.
- Children as young as five worked in coal mines.
- Meanwhile, factory owners grew spectacularly rich.

The Gap Widens

The gap between capital and labour had never been wider. This was the world that **Karl Marx** (1818–1883) set out to explain.

Karl Marx and Friedrich Engels

- **Karl Marx** (1818–1883): German philosopher, economist, and revolutionary.
- Collaborated with **Friedrich Engels**, son of a Manchester factory owner who witnessed working conditions first-hand.
- *The Communist Manifesto* (1848): "The history of all hitherto existing society is the history of class struggles."
- *Capital* (Das Kapital, 1867): a massive analysis of how capitalism works — and why, Marx argued, it would eventually destroy itself.

Method

Writing from the reading room of the British Museum, Marx combined Hegelian philosophy, Ricardo's economics, and detailed empirical study of factory reports.

Surplus Value

"The essential difference between the various economic forms of society . . . lies only in the mode in which this surplus labour is in each case extracted from the actual producer, the labourer."

— Karl Marx, *Capital*, Vol. I, Ch. 9

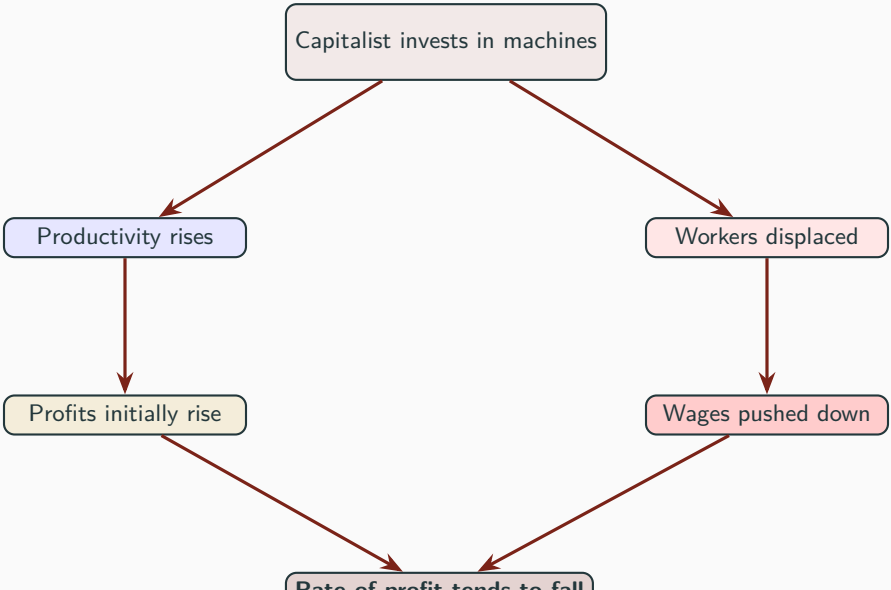
Labour Theory of Value

The value of a good comes from the labour required to produce it. A chair is worth more than a stick because more labour went into it.

Surplus Value

Workers produce more value than they receive in wages. A worker produces £10/hour but earns £3. The remaining £7 is **surplus value**, captured as profit.

The Machinery Question



The Machinery Question in the 21st Century

Marx's question — *does technology help or hurt workers?* — is more relevant today than ever.

- Since the 1980s, the **labour share of income** has been declining across developed economies.
- Automation, globalisation, and now artificial intelligence raise the same concerns Marx identified in 1867.
- We cannot resolve this debate in one module, but we can build a model that **formalises the question** and lets us explore the forces at work.

Central Question

Under what conditions does capital accumulation reduce workers' share of income — and under what conditions does it raise their wages?

Edinburgh Connection

Edinburgh's own Industrial Revolution was the backdrop for these debates:

- The city was once Britain's **second-largest brewing centre**.
- Expansion of **printing** and **engineering** industries.
- The *Edinburgh Review* (founded 1802) was one of the key journals where political economists debated the effects of machinery on employment.

The Edinburgh Review

Founded by Sydney Smith, Francis Jeffrey, and Henry Brougham, the *Edinburgh Review* shaped public debate on industrialisation, free trade, and the condition of the working class for over a century.

Part II — The Computation

Setting Up

```
import numpy as np
import matplotlib.pyplot as plt
```

- `numpy` — numerical arrays and operations
- `matplotlib` — plotting

Today we model how output is **divided** between capital and labour — and what happens when capitalists accumulate more and more machines.

The Cobb-Douglas Production Function

We model output using a **Cobb-Douglas production function**:

$$Y = A \cdot K^{\alpha} \cdot L^{1-\alpha}$$

K	Capital (machines, factories)
L	Labour (workers)
A	Total factor productivity
$\alpha \in (0, 1)$	The capital share — the fraction of income going to capital owners

Key property: **constant returns to scale**. Doubling both K and L exactly doubles Y.

Factor Payments: Who Gets What?

In competitive markets, factors are paid their **marginal products**:

$$\text{Wage:} \quad w = \text{MPL} = (1-\alpha)\frac{Y}{L} = (1-\alpha)\,A\,K^{\alpha}\,L^{-\alpha}$$

$$\text{Rental rate:} \quad r = \text{MPK} = \alpha\frac{Y}{K} = \alpha\,A\,K^{\alpha-1}\,L^{1-\alpha}$$

Labour Share

$$\frac{wL}{Y} = 1-\alpha$$

Capital Share

$$\frac{rK}{Y} = \alpha$$

Under Cobb-Douglas, these shares are constant — a result roughly true from 1950 to 1980, but since broken down.

Defining the Production Function in Python

```
# Parameters
A = 1.0
alpha = 0.33     # capital share
L = 100          # fixed labour supply

def output(K, L=L, A=A, alpha=alpha):
    return A * K**alpha * L**(1 - alpha)

def wage(K, L=L, A=A, alpha=alpha):
    return (1 - alpha) * A * K**alpha * L**(-alpha)

def rental_rate(K, L=L, A=A, alpha=alpha):
    return alpha * A * K**(alpha - 1) * L**(1 - alpha)
```

As capital accumulates:

- Output rises (but with diminishing returns)
- Wages **rise** (more capital per worker)
- Return on capital **falls** (diminishing returns)
- Labour share stays constant at $1 - \alpha = 67\%$

Output and Factor Prices as Capital Grows

Under Cobb-Douglas, wages **rise** with capital but the **shares** remain fixed: $wL/Y = 1 - \alpha$, $rK/Y = \alpha$.

Plotting Factor Prices in Python

```
K_range = np.linspace(10, 500, 200)
Y = output(K_range)
total_wages = wage(K_range) * L
total_profits = rental_rate(K_range) * K_range

fig, axes = plt.subplots(1, 3, figsize=(15, 4.5))

axes[0].plot(K_range, Y, color='seagreen', linewidth=2)
axes[0].set_title('Total Output')

axes[1].plot(K_range, wage(K_range), label='Wage per worker')
axes[1].plot(K_range, rental_rate(K_range), label='Return on capital')
axes[1].legend()

axes[2].plot(K_range, total_wages, label='Total wages (wL)')
axes[2].plot(K_range, total_profits, label='Total profits (rK)')
axes[2].legend()
```

Capital Accumulation Dynamics

Marx was interested in the *dynamics* of capitalism. Capitalists reinvest their profits, accumulating more and more capital:

$$K_{t+1} = (1 - \delta)\, K_t + s \cdot r_t \cdot K_t$$

- δ Depreciation rate (machines wear out)
- s Fraction of profits that capitalists reinvest
- $r_t \cdot K_t$ Total profits in period t

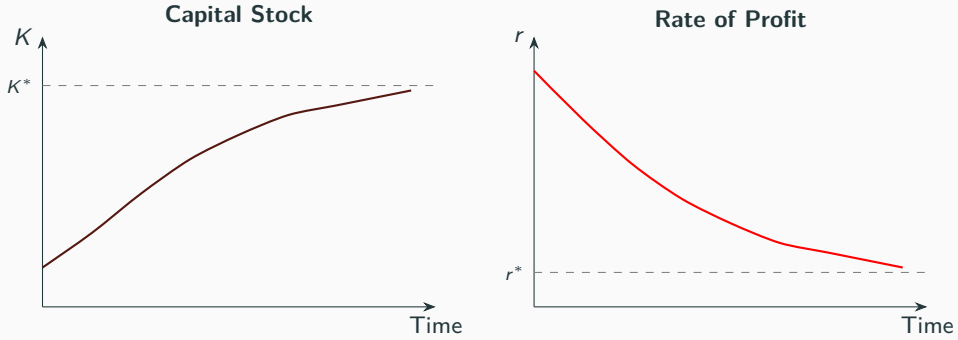
Steady State

In the long run, K converges to a steady state where investment equals depreciation: $s \cdot r \cdot K = \delta \cdot K$, giving $r_{ss} = \delta / s$.

Simulating Capital Accumulation

```
T = 200
delta = 0.05     # depreciation rate
s = 0.80         # reinvestment rate
K0 = 50          # initial capital

def simulate_accumulation(K0, L, A, alpha, delta, s, T):
    K = np.empty(T)
    K[0] = K0
    Y_t, w_t, r_t = [np.empty(T) for _ in range(3)]
    for t in range(T):
        Y_t[t] = output(K[t], L, A, alpha)
        w_t[t] = wage(K[t], L, A, alpha)
        r_t[t] = rental_rate(K[t], L, A, alpha)
        if t < T - 1:
            K[t+1] = (1 - delta)*K[t] + s*r_t[t]*K[t]
    return K, Y_t, w_t, r_t

K_sim, Y_sim, w_sim, r_sim = simulate_accumulation(
    K0, L, A, alpha, delta, s, T)
```

Convergence: Capital and the Rate of Profit



As capital accumulates, **diminishing returns** drive down the rate of profit — exactly Marx's "tendency of the rate of profit to fall."

But wages **rise**: more capital per worker raises productivity.

Marx's Falling Rate of Profit

- Marx predicted that as capitalists invest, the return on each additional unit of capital declines.
- In our model: $r = \alpha A K^{\alpha-1} L^{1-\alpha}$. As $K \to \infty$, $r \to 0$.
- At steady state: $r_{ss} = \delta / s$.
- With $\delta = 0.05$, $s = 0.80$: $r_{ss} = 6.25\%$.

What Marx Missed

Sustained **technological progress** (A rising) can offset diminishing returns and prevent the profit rate from collapsing. Marx did not anticipate continuous innovation.

Key Insight

Diminishing returns alone push r down, but productivity growth can **counteract** this tendency indefinitely.

Beyond Cobb-Douglas: The CES Production Function

The Cobb-Douglas gives a **constant** labour share — which does not match post-1980 data. We need a more flexible model:

$$Y = A\left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha) L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

The parameter σ is the **elasticity of substitution** — how easily capital can replace labour.

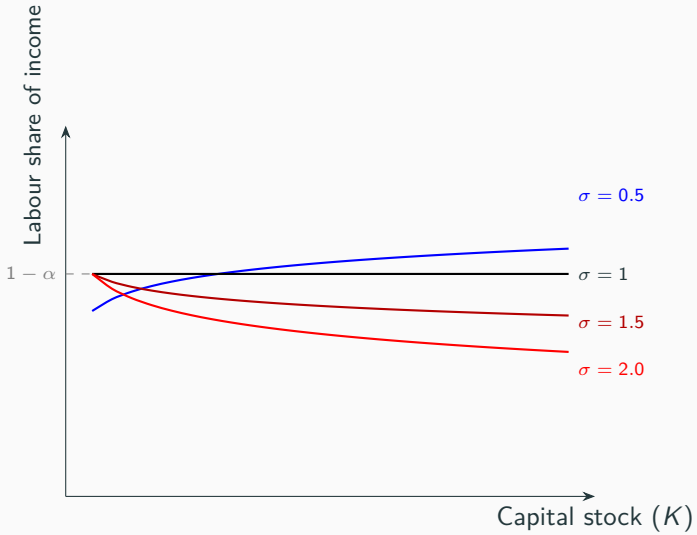
$\sigma = 1$	$\sigma > 1$	$\sigma < 1$
Cobb-Douglas. Labour share constant.	Easy to substitute. Labour share falls as K grows.	Complements. Labour share **rises** as K grows.

CES Production Function in Python

```python
def ces_output(K, L, A, alpha, sigma):
    """CES production function."""
    rho = (sigma - 1) / sigma
    return A * (alpha * K**rho + (1 - alpha) * L**rho)**(1/rho)

def ces_labour_share(K, L, A, alpha, sigma):
    """Labour share under CES production."""
    rho = (sigma - 1) / sigma
    Y = ces_output(K, L, A, alpha, sigma)
    inner = alpha * K**rho + (1 - alpha) * L**rho
    MPL = A * (1 - alpha) * L**(rho - 1) * inner**(1/rho - 1)
    return MPL * L / Y
```

The key idea: when $\sigma > 1$ and K rises, the marginal product of labour falls *faster* than the capital–labour ratio rises, so workers' share of income **declines**.

Labour Share Under Different Elasticities

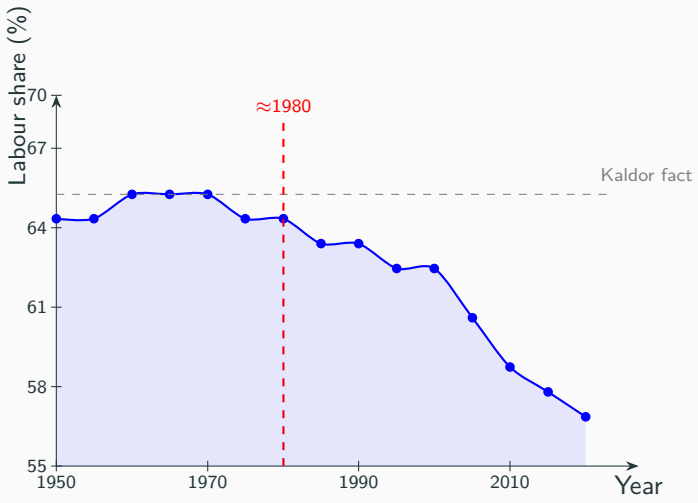


Computing Labour Shares for Different σ

```
K_range = np.linspace(50, 1000, 200)

fig, ax = plt.subplots(figsize=(9, 5))

for sigma, color, ls in [
        (0.5, 'steelblue', 'Complements'),
        (1.01, 'black', 'Cobb-Douglas'),
        (1.5, 'coral', 'Substitutes'),
        (2.0, 'red', 'Easy substitution')]:
    shares = [ces_labour_share(K, L, A, 0.33, sigma)
              for K in K_range]
    ax.plot(K_range, shares, color=color, linewidth=2,
            label=f'{ls} (sigma={sigma})')

ax.set_xlabel('Capital stock (K)')
ax.set_ylabel('Labour share of income')
ax.legend()
ax.set_ylim(0, 1)
```

Recent estimates put σ between 1.2 and 1.6 for the US economy.

US Labour Share, 1950–2020



For decades, the labour share was ≈ 66% — a "Kaldor fact." Since ≈1980, it has declined by

Plotting the Labour Share Data

```
# Stylised US labour share data
# (based on BLS and Karabarbounis & Neiman 2014)
years = np.array([1950, 1955, 1960, 1965, 1970, 1975, 1980,
                  1985, 1990, 1995, 2000, 2005, 2010, 2015, 2020])
labour_share_us = np.array([0.65, 0.65, 0.66, 0.66, 0.66, 0.65,
    0.65, 0.64, 0.64, 0.63, 0.63, 0.61, 0.59, 0.58, 0.57])

fig, ax = plt.subplots(figsize=(9, 5))
ax.plot(years, labour_share_us*100, 'o-', linewidth=2)
ax.axvline(1980, color='red', linestyle='--', alpha=0.4,
           label='c. 1980: trend break')
ax.axhline(66, color='gray', linestyle=':', alpha=0.5,
           label='"Kaldor fact"')
ax.set_title('US Labour Share of Income, 1950-2020')
ax.legend()
```

Marx might say: "I told you so."

Part III — Exercises & Discussion

Exercise 1: Marx's Falling Rate of Profit

(a) Simulate capital accumulation for 300 periods using Cobb-Douglas ($\alpha = 0.33$, $L = 100$, $A = 1$, $\delta = 0.05$, $s = 0.8$, $K_0 = 50$). Plot the rate of profit r_t over time. Does it fall?

(b) Compute the steady-state capital stock. Show that $r_{ss} = \delta / s$.

(c) Add technological progress: $A_t = A_0 \cdot e^{0.01\,t}$. Simulate again. Does the rate of profit still fall to the same level?

(d) In 2–3 sentences, assess Marx's prediction in light of your simulations.

Key Insight

Marx ignored sustained technological progress — does it save capitalism from the falling profit rate?

Exercise 2: Automation and the CES Function

Suppose the elasticity of substitution increases from $\sigma = 1$ to $\sigma = 1.5$ due to automation (robots can do jobs previously done by humans).

(a) With $K = 200$, $L = 100$, $A = 1$, $\alpha = 0.33$, compute the labour share for $\sigma = 0.8,\ 1.0,\ 1.2,\ 1.5,\ 2.0$. Present as a table.

(b) For $\sigma = 1.5$, simulate capital accumulation over 200 periods ($K_0 = 50$, $s = 0.3$, $\delta = 0.05$). Plot the labour share over time.

(c) At what level does the labour share stabilise? How does this compare to the Cobb-Douglas case?

Exercise 3: Was Marx Right About Wages?

Marx predicted wages would be pushed to subsistence. The evidence is more nuanced: real wages have risen enormously since 1850, but wages as a *share* of income have fallen recently.

(a) Using the Cobb-Douglas simulation, plot *real wages* w_t alongside the *labour share* $w_t L / Y_t$ over time. Can wages rise while the labour share stays constant?

(b) Add population growth: $L_t = L_0 \cdot e^{0.005\,t}$. Simulate capital accumulation and wages over 200 periods. Do wages still rise?

(c) In 2–3 sentences, explain where Marx was right and where he was wrong about wages.

Quiz

Quiz: Conceptual Questions (1/2)

Q1. Marx's "surplus value" refers to:

(a) The price markup that firms charge over cost

(b) The difference between the value workers produce and the wages they receive

(c) Excess inventory that goes unsold

(d) Government tax revenue

Q2. The "machinery question" in classical economics asks:

(a) Whether steam engines are more efficient than waterwheels

(b) Whether technological change benefits or harms workers

(c) How to optimally maintain factory equipment

(d) Whether agriculture or manufacturing drives growth

Quiz: Conceptual Questions (2/2)

Q3. Marx predicted the rate of profit would tend to fall because:

(a) Competition would drive prices down
(b) As more capital accumulates, diminishing returns reduce the return on each unit
(c) Workers would demand higher wages
(d) Government regulation would squeeze profits

Q4. The labour share of income is defined as:

(a) The number of workers divided by total population
(b) Total wages divided by GDP
(c) Average wage divided by average productivity
(d) Employment rate times hours worked

Q5. Since 1980, the labour share in developed countries has:

(a) Remained constant (a "Kaldor fact")
(b) Risen steadily

Quiz: Computational Questions (1/2)

Q6. Under $Y = AK^{0.33}L^{0.67}$, the labour share equals:

(a) 0.33
(b) 0.67
(c) It depends on K and L
(d) 0.50

Q7. In the CES function, if $\sigma > 1$ and capital increases, the labour share:

(a) Rises
(b) Falls
(c) Stays constant
(d) First rises then falls

Q8. Under Cobb-Douglas, if capital doubles while labour is fixed, wages:

(a) Stay the same
(b) Rise by a factor of 2^{α}

Quiz: Computational Questions (2/2)

Q9. The elasticity of substitution σ measures:

(a) How fast capital depreciates
(b) How easily firms can replace labour with capital
(c) The rate of technological progress
(d) The speed of wage adjustment

Q10. Under Cobb-Douglas with capital accumulation, real wages actually:

(a) Fall, confirming Marx
(b) Rise, as more capital makes each worker more productive
(c) Stay constant at subsistence
(d) Depend entirely on union bargaining power

Answers

Q1: (b) Q2: (b) Q3: (b) Q4: (b) Q5: (c)
Q6: (b) Q7: (b) Q8: (b) Q9: (b) Q10: (b)

Key Takeaways

What We Learned This Week

1. Marx analysed capitalism as a system that generates **surplus value** — the gap between what workers produce and what they are paid.
2. The **machinery question** — does technology help or hurt workers? — remains central to modern economics.
3. Under **Cobb-Douglas**, capital accumulation raises wages but the labour share stays constant at $1 - \alpha$.
4. The **CES production function** allows the labour share to change: when $\sigma > 1$, capital accumulation reduces the labour share.
5. The **US labour share** has fallen $\approx$10 pp since 1980 — consistent with $\sigma > 1$.
6. Marx predicted the **falling rate of profit** (diminishing returns to capital), but missed the role of sustained technological progress.

Further Reading

- Marx, K. *Capital*, Volume I (1867), Chs. 1, 7, 15 (selections).
- Heilbroner, R. *The Worldly Philosophers*, Ch. 6 ("The Inexorable System of Karl Marx").
- Piketty, T. *Capital in the Twenty-First Century* (2014), Ch. 6.
- Karabarbounis, L. & Neiman, B. "The Global Decline of the Labor Share" (*QJE*, 2014).

Next week: The Marginalist Revolution — how economics shifted from class-based analysis to individual optimisation in the 1870s.