Module 8 — Expectations and the Lucas Critique

Friedman, Lucas, and Why Policy Rules Matter

Dr Juan Zurita

From Smith to Simulation
The University of Edinburgh · School of Economics

Part I — The History

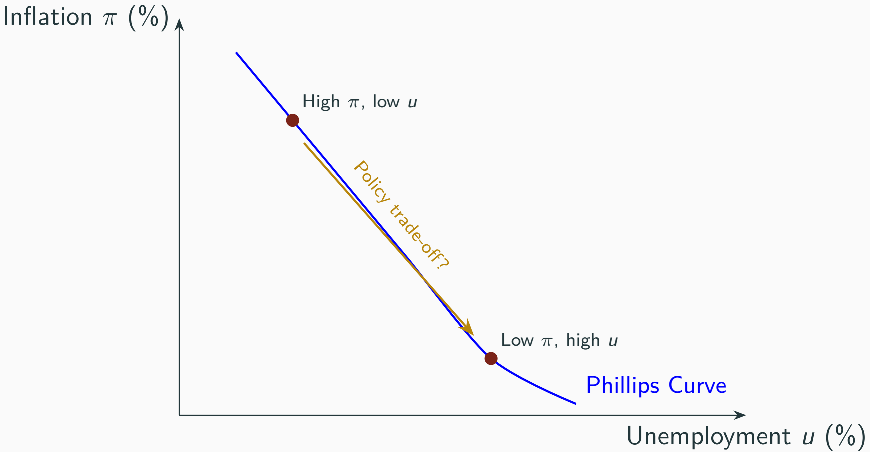
The Phillips Curve and the Keynesian Consensus

- 1958: A. W. Phillips discovers a stable inverse relationship between wage inflation and unemployment in UK data (1861–1957).
- Samuelson and Solow recast it as a trade-off between *price* inflation and unemployment.
- The implication: policymakers can choose their preferred point on the curve.

The 1960s Dream

Want lower unemployment? Accept a bit more inflation. Want price stability? Tolerate higher joblessness. Macroeconomic policy became a matter of **calibrating aggregate demand**.

The Naive Phillips Curve



The apparent “menu of options” that guided policy in the 1960s.

Friedman's Presidential Address (1968)

Milton Friedman (1912–2006), leader of the **Chicago School** and champion of **monetarism**:

- The Phillips curve trade-off is an illusion built on **money illusion**.
- Workers care about *real* wages, not nominal ones.
- Any attempt to push unemployment below the **"natural rate"** succeeds only *temporarily*.

Natural Rate

The unemployment rate consistent with stable inflation — the rate that prevails when expectations are correct and labour markets are in equilibrium (accounting for search, matching, and structural frictions).

Friedman's Logic: The Accelerationist Argument

1. Government expands demand, pushing up prices.
2. Workers mistake higher nominal wages for higher *real* wages $\Rightarrow$ supply more labour.
3. Unemployment falls *temporarily* below u^*.
4. Workers catch on: they revise inflation expectations **upward**.
5. Real wages return to equilibrium; unemployment returns to u^*, but now inflation is *higher*.
6. To get the same temporary boost, the government must generate even more inflation — an accelerating spiral.

The Friedman–Phelps Hypothesis

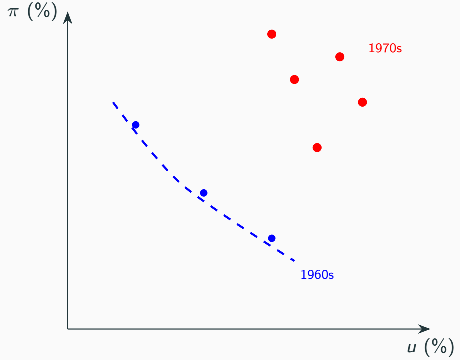
In the **long run**, the Phillips curve is *vertical* at the natural rate. There is no permanent trade-off.

Stagflation: Theory Meets Reality

The 1970s delivered devastating confirmation:

- Oil price shocks of 1973 and 1979.
- Loose monetary policy.
- Stagflation: simultaneously high inflation *and* high unemployment.
- The naive Phillips curve said this **could not happen**.

Data points of the 1970s scattered across the inflation–unemployment space, destroying the tidy regularity of the 1960s.



Stylised: 1970s points in the "impossible" quadrant.

Robert Lucas and Rational Expectations

Robert Lucas (1937–2023), University of Chicago.

If agents are *rational*, why form expectations by mechanically looking backward?

- Agents use **all available information**, including knowledge of government policy rules.
- Forecasts may err in any period, but are *unbiased*:

$$\pi_t^e = E[\pi_t \mid \mathcal{I}_{t-1}]$$

- Nobel Prize in Economics, 1995.

Policy Ineffectiveness

A *systematic* demand expansion has **no real effect**, even in the short run. Agents anticipate the inflation and adjust. Only surprises move real variables — and surprises are unsustainable as policy.

(Sargent & Wallace, 1975)

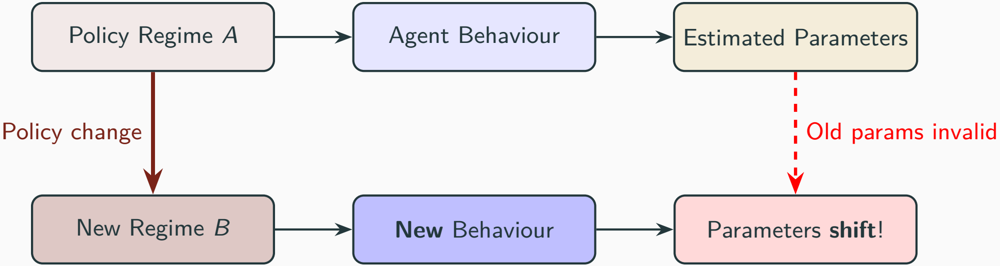
The Lucas Critique (1976)

"Given that the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models."

— Robert Lucas, 1976

- Parameters of econometric models are **not structural constants**.
- They are reduced-form coefficients that depend on the policy regime.
- Change the policy $\Rightarrow$ agents change behaviour $\Rightarrow$ parameters shift.
- A model estimated under one regime **cannot predict outcomes** under a different regime.

The Lucas Critique: A Diagram

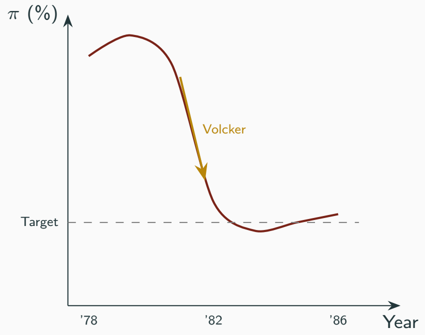


This forced a methodological revolution toward **microfounded models** (DSGE) whose deep parameters (preferences, technology) remain invariant to policy changes.

The Volcker Disinflation

- 1979: **Paul Volcker** appointed Fed Chairman.
- Abandoned interest-rate targeting; targeted monetary aggregates.
- Deliberately induced severe recession to break inflation expectations.
- Unemployment peaked above 10%.
- Inflation fell from double digits to $\approx 4\%$ by 1983.

Demonstrated both the **power of expectations** (once credibly anchored, inflation fell) and the cost of re-anchoring them.



Stylised US inflation path, 1978–1986.

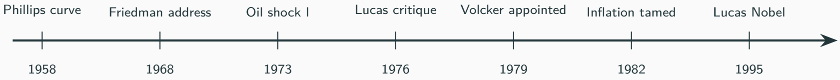
Edinburgh Connection

Theory and Empirics Meet

The University of Edinburgh's tradition — from David Hume's monetary theory to modern theoretical and applied econometrics — is precisely the synthesis the expectations revolution demanded.

- The debate was never purely theoretical — driven by **empirical failures** (breakdown of the Phillips curve).
- Never purely empirical — required **deep theoretical innovation** (rational expectations, the Lucas critique).
- Edinburgh trains economists who move fluently between formal theory, computational modelling, and careful empirical work — exactly the skill set the expectations revolution required.

Timeline: The Expectations Revolution



From stable trade-off to intellectual revolution in under two decades.

The lesson: **people are not passive objects of policy.** They think, anticipate, and adapt.

Part II — The Computation

Setting Up

```
import numpy as np
import matplotlib.pyplot as plt

# Reproducibility
np.random.seed(42)
```

- `numpy` — numerical arrays, random number generation
- `matplotlib` — plotting
- We will simulate inflation processes, expectation formation, and policy experiments

The Original Phillips Curve

The naive Phillips curve posits a simple inverse relationship:

$$\pi = \alpha - \beta\,(u - u^*)$$

where u^* is the natural rate of unemployment.

Parameters

$\alpha = 3.0$ (baseline inflation at $u = u^*$)
$\beta = 1.2$ (slope)
$u^* = 5.0\%$ (natural rate)

Interpretation

At $u = 3\%$: $\pi = 3 + 1.2(2) = 5.4\%$
At $u = 7\%$: $\pi = 3 - 1.2(2) = 0.6\%$
The "menu of options"

The AR(1) Process for Inflation

Inflation is often modelled as a first-order autoregressive process:

$$\pi_t = a + \rho\,\pi_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma^2)$$

Key properties (when $|\rho| < 1$):

- Stationary
- Unconditional mean: $\mu = \dfrac{a}{1-\rho}$
- Unconditional variance: $\sigma_\pi^2 = \dfrac{\sigma^2}{1-\rho^2}$
- Autocorrelation at lag k: ρ^k

Special cases:

- $\rho = 0$: white noise
- $\rho = 1$: random walk (non-stationary)
- $|\rho| > 1$: explosive
- High ρ: persistent, slow mean-reversion

Simulating an AR(1) Process

```
def simulate_ar1(a, rho, sigma, T, pi_0=None):
    """Simulate AR(1): pi_t = a + rho * pi_{t-1} + eps_t."""
    if pi_0 is None:
        pi_0 = a / (1 - rho) if abs(rho) < 1 else 0.0
    pi = np.zeros(T)
    pi[0] = pi_0
    eps = np.random.normal(0, sigma, T)
    for t in range(1, T):
        pi[t] = a + rho * pi[t-1] + eps[t]
    return pi

# Parameters
T = 200; a = 0.5; rho = 0.85; sigma = 0.5
pi_ar1 = simulate_ar1(a, rho, sigma, T)

# Theoretical moments
mu_theory = a / (1 - rho)     # = 3.333
var_theory = sigma**2 / (1 - rho**2)  # = 0.948
```

Adaptive Expectations

Under **adaptive expectations**, agents look *backward*:

$$\pi_t^e = \pi_{t-1}$$

- Simple rule: expect tomorrow's inflation to equal today's.
- When inflation is rising, adaptive expectations systematically underpredict.
- When inflation is falling, they systematically overpredict.
- Forecast errors are **serially correlated** — a rational agent should be able to do better.

Key Weakness

Adaptive expectations leave **money on the table**: predictable patterns in forecast errors mean the agent is not using all available information.

Adaptive Expectations: Code

```
pi_actual = pi_ar1.copy()

# Adaptive: expected = last period's actual
pi_e_adaptive = np.zeros(T)
pi_e_adaptive[0] = pi_actual[0]
for t in range(1, T):
    pi_e_adaptive[t] = pi_actual[t - 1]

# Forecast errors
fe_adaptive = pi_actual - pi_e_adaptive

print(f"Mean forecast error:  {fe_adaptive.mean():.4f}")
print(f"Std forecast error:   {fe_adaptive.std():.4f}")
```

The adaptive forecast tracks the actual series but with a **lag** — always one step behind any systematic trend.

Rational Expectations

Under **rational expectations**, agents use *all available information*:

$$\pi_t^e = E[\pi_t \mid \mathcal{I}_{t-1}]$$

If inflation follows $\pi_t = a + \rho\,\pi_{t-1} + \varepsilon_t$ and agents know a, ρ, and observe π_{t-1}:

$$\pi_t^e = a + \rho\,\pi_{t-1}$$

- Forecast error: $\pi_t - \pi_t^e = \varepsilon_t$ — pure white noise.
- **Mean-zero** and **serially uncorrelated**.
- No information at time $t-1$ can improve the forecast.

Not Clairvoyance

Rational expectations $\neq$ perfect foresight. Agents make errors, but the errors are **unpredictable**. No systematic pattern remains to exploit.

Rational Expectations: Code

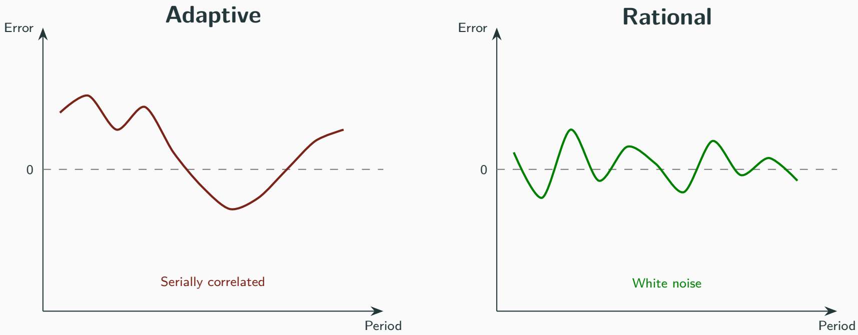
```
# Rational: conditional expectation of AR(1)
pi_e_rational = np.zeros(T)
pi_e_rational[0] = mu_theory  # unconditional mean
for t in range(1, T):
    pi_e_rational[t] = a + rho * pi_actual[t - 1]

# Forecast errors
fe_rational = pi_actual - pi_e_rational

print(f"Mean forecast error:  {fe_rational.mean():.4f}")
print(f"Std forecast error:   {fe_rational.std():.4f}")

# Serial correlation of errors
from numpy import corrcoef
ac_adapt = corrcoef(fe_adaptive[1:], fe_adaptive[:-1])[0,1]
ac_ratio = corrcoef(fe_rational[1:], fe_rational[:-1])[0,1]
print(f"Autocorr (adaptive):  {ac_adapt:.4f}")   # high
print(f"Autocorr (rational):  {ac_ratio:.4f}")   # near zero
```

Comparing Forecast Errors



Adaptive: Errors show patterns.
A rational agent could exploit them.

Rational: Errors are unpredictable.
No further improvement possible.

The Policy Experiment

The heart of the matter: can the government exploit the Phillips curve?

Expectations-augmented Phillips curve:

$$\pi_t = \pi_t^e - \beta(u_t - u^*) + \varepsilon_t$$

Policy rule: after period 10, government injects extra inflation $\delta = 2$ percentage points per period.

Under Adaptive

$\pi_t^e = \pi_{t-1}$. The surprise $\pi_t - \pi_t^e = \delta + \varepsilon_t > 0$ pushes $u_t < u^*$. But inflation **ratchets up** each period.

Under Rational

Agents *know* the policy rule: $\pi_t^e = \pi_{t-1} + \delta$. Surprise $= \varepsilon_t$ only. $u_t = u^* + \text{noise}$. **No real effect.**

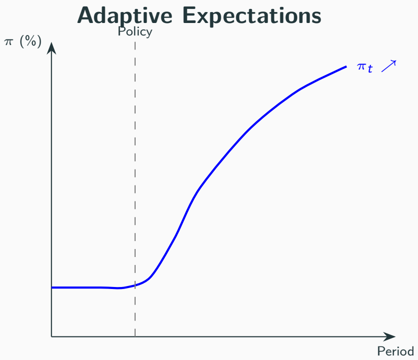
Policy Experiment: Code

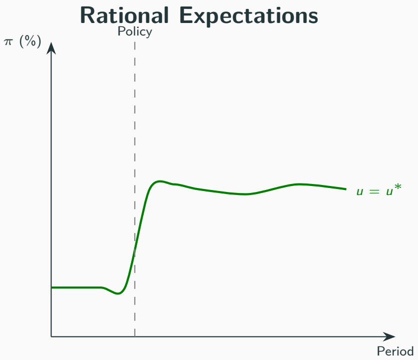
```
T_pol = 60; u_nat = 5.0; beta_pc = 1.0; delta = 2.0
policy_start = 10
eps = np.random.normal(0, 0.3, T_pol)

# --- Adaptive ---
pi_a, pi_e_a, u_a = np.zeros(T_pol), np.zeros(T_pol), np.full(T_pol, u_nat)
pi_a[0] = pi_e_a[0] = 2.0
for t in range(1, T_pol):
    pi_e_a[t] = pi_a[t-1]
    pi_a[t] = pi_e_a[t] + (delta if t>=policy_start else 0) + eps[t]
    u_a[t] = u_nat - (pi_a[t] - pi_e_a[t] - eps[t]) / beta_pc

# --- Rational ---
pi_r, pi_e_r, u_r = np.zeros(T_pol), np.zeros(T_pol), np.full(T_pol, u_nat)
pi_r[0] = pi_e_r[0] = 2.0
for t in range(1, T_pol):
    pi_e_r[t] = pi_r[t-1] + (delta if t>=policy_start else 0)
    pi_r[t] = pi_e_r[t] + eps[t]
    u_r[t] = u_nat - (pi_r[t] - pi_e_r[t] - eps[t]) / beta_pc
```

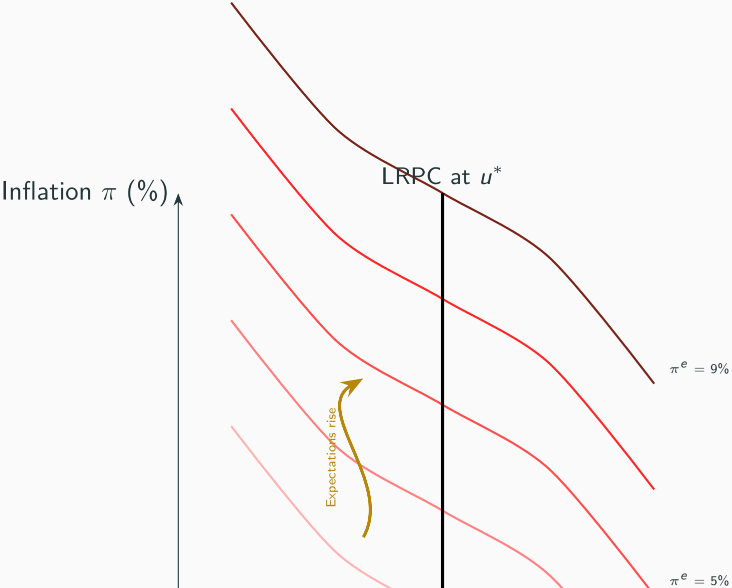
Policy Experiment: Results





- **Adaptive**: unemployment drops temporarily, but inflation accelerates without bound. Friedman's prediction.
- **Rational**: unemployment stays at u^*. Policy only raises inflation. Lucas's result.

The Augmented Phillips Curve



Part III — Exercises & Discussion

Exercise 1: AR(1) Persistence and Stationarity

Explore how the persistence parameter ρ shapes an AR(1) process.

1. Simulate four AR(1) processes ($T = 300$, $a = 0.5$, $\sigma = 0.5$) with $\rho \in \{0.2,\ 0.7,\ 0.95,\ 1.0\}$. Use $\pi_0 = 0$. Plot on a 2×2 grid.
2. For stationary cases, compare theoretical and sample moments: $\mu = a/(1-\rho), \quad \sigma_\pi = \sigma/\sqrt{1-\rho^2}$.
3. Compute sample autocorrelation at lags $k = 1, \ldots, 20$. Compare with theoretical ρ^k.
4. Why is $\rho = 1$ non-stationary? When might inflation have a unit root?

Key Insight

High ρ means shocks are persistent; $\rho = 1$ means shocks are **permanent**. Credible monetary policy makes inflation stationary.

Exercise 1: Starter Code

```python
T_ex1 = 300; a_ex1 = 0.5; sigma_ex1 = 0.5
rhos = [0.2, 0.7, 0.95, 1.0]
np.random.seed(2024)

fig, axes = plt.subplots(2, 2, figsize=(14, 8), sharex=True)
axes = axes.flatten()

for i, rho_i in enumerate(rhos):
    pi_i = simulate_ar1(a_ex1, rho_i, sigma_ex1, T_ex1, pi_0=0.0)
    axes[i].plot(pi_i, linewidth=0.8)
    axes[i].set_title(f'$\\rho = {rho_i}$')
    if rho_i < 1:
        mu_i = a_ex1 / (1 - rho_i)
        axes[i].axhline(y=mu_i, color='red', linestyle='--',
                        label=f'$\\mu = {mu_i:.2f}$')
        axes[i].legend()

plt.tight_layout()
plt.show()
```

Exercise 2: The Lucas Critique in Action

A central bank sets inflation via a policy rule:

$$\pi_t = \bar{\pi} + \gamma \left(u_{t-1} - u^* \right)$$

with $\bar{\pi} = 2\%$, $\gamma = -0.8$, $u^* = 5\%$. Unemployment obeys:

$$u_t = u^* - \alpha_{pc} \left(\pi_t - \pi_t^e \right) + \eta_t$$

(a) Simulate under **adaptive** expectations ($\pi_t^e = \pi_{t-1}$).

(b) Simulate under **rational** expectations ($\pi_t^e = \bar{\pi} + \gamma(u_{t-1} - u^*)$).
Show that $\pi_t - \pi_t^e = 0 \Rightarrow$ no systematic effect.

(c) Change the policy to $\gamma = -1.6$. Compare outcomes.

(d) Explain: why can an econometrician not use estimates from (a) to predict (c)?

Exercise 2: Starter Code

```
def simulate_lucas_critique(gamma, exp_type='adaptive',
                            T=100, seed=99):
    np.random.seed(seed)
    eta = np.random.normal(0, 0.3, T)
    pi = np.zeros(T); pi_e = np.zeros(T)
    u = np.full(T, 5.0)
    pi[0] = pi_e[0] = 2.0; u[0] = 5.0 + eta[0]
    for t in range(1, T):
        pi[t] = 2.0 + gamma * (u[t-1] - 5.0)
        if exp_type == 'adaptive':
            pi_e[t] = pi[t - 1]
        else:  # rational
            pi_e[t] = 2.0 + gamma * (u[t-1] - 5.0)
        u[t] = 5.0 - 0.5 * (pi[t] - pi_e[t]) + eta[t]
    return pi, pi_e, u

pi_a, _, u_a = simulate_lucas_critique(-0.8, 'adaptive')
pi_r, _, u_r = simulate_lucas_critique(-0.8, 'rational')
```

Exercise 3: Disinflation — Cold Turkey vs. Gradualism

A Volcker-style disinflation: reduce π from 10% to 2%.

Cold Turkey

Immediately set $\pi_t = 2\%$ for all $t \geq 1$.

Fast but potentially very painful.

Gradualism

Reduce inflation linearly over 20 periods:

$\pi_t = \max(2,\ 10 - 0.4t)$.

Slower, spread the cost.

(a) Simulate both under **adaptive** expectations. Plot π_t and u_t.

(b) Repeat under **rational** expectations (full credibility).

(c) Compute the **sacrifice ratio**: cumulative excess unemployment per percentage point of disinflation. Compare in a bar chart.

(d) Why is disinflation costly under adaptive but costless under rational expectations? What does this say about **credibility**?

Exercise 3: Starter Code

```python
T_ex3 = 50; pi_0_ex3 = 10.0; pi_target = 2.0
np.random.seed(777)
eta = np.random.normal(0, 0.2, T_ex3)

def disinflation_path(strategy, T):
    pi_cb = np.zeros(T); pi_cb[0] = pi_0_ex3
    for t in range(1, T):
        if strategy == 'cold_turkey':
            pi_cb[t] = pi_target
        else:  # gradual
            pi_cb[t] = max(pi_target,
                           pi_0_ex3 - (pi_0_ex3-pi_target)*t/20)
    return pi_cb

def sacrifice_ratio(u, u_star=5.0):
    return np.sum(np.maximum(u - u_star, 0)) / (10 - 2)
```

Key Takeaways

What We Learned This Week

1. The **naive Phillips curve** offered an apparent trade-off between inflation and unemployment, but it was an illusion.
2. **Friedman** (1968) showed the trade-off is temporary: in the long run, the Phillips curve is *vertical* at the natural rate.
3. **Lucas** took this further with **rational expectations**: systematic policy has no real effect because agents anticipate it.
4. The **Lucas critique** invalidated old-style econometric policy evaluation — parameters shift when regimes change.
5. **Credibility** is the crucial variable: disinflation is costly when expectations must be re-anchored, as the Volcker episode showed.
6. We can **simulate** all of this: AR(1) processes, adaptive vs. rational expectations, and policy experiments that demonstrate these ideas computationally.

Further Reading

- Friedman, M. "The Role of Monetary Policy," *American Economic Review*, 1968.
- Lucas, R. "Econometric Policy Evaluation: A Critique," *Carnegie-Rochester Conference Series*, 1976.
- Sargent, T. and N. Wallace. "Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule," *Journal of Political Economy*, 1975.
- Mankiw, N. G. *Macroeconomics*, Chs. 14–15.
- Romer, D. *Advanced Macroeconomics*, Ch. 6.

Next week: The New Keynesian Synthesis —
rational expectations meets price rigidities,
and why credible institutions matter for monetary policy.